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ABSTRACT

The material conservation laws are derived for the axisymmetric flows of the inviscid barotropic gas inside the mushroom clouds. The invariant functions $\Psi(\mu)$ and $\Psi_G(\mu)$ of an independent variable μ are constructed for any pure poloidal compressible gas flow.

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I. INTRODUCTION

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incompressible fluid with variable density $\rho(r, z, t)$, the functions

[redacted]

are closely related with the geometry of surfaces $\mathcal{H}(r,z,t)$ = const. In Sec. III, we demonstrate that functions $\Psi(\mu)$ and $\Psi_G(\mu)$ are linked by certain differential equations.

[redacted]

(3) (4) (5)

[redacted]

(2)

[redacted]

(1) 5

[redacted]

=

[redacted]

(8) (10)

[redacted]

=

Using this formula, we find after differentiation and cancellation of similar terms the following:

[redacted]

(8)

(3) (4)

[redacted]

(1) (11) (14)

[redacted]

=

[redacted]

=

Substituting expression (20) into Eq. (17) and canceling similar terms, we get

Remark 3. Equation (23) implies that the diffeomorphisms defined by the axisymmetric pure poloidal gas flows preserve the valsurface $K_\mu^2(t)$ are frozen into the flow, in the standard terminology.

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provided that the denominator $|\nabla\mathcal{H}(\mathbf{x},t)|$ in the integral (34) is non-

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$$\Psi_G(\mu) = \int_{\mu_1}^{\mu} G(\xi)\frac{d\Psi(\xi)}{d\xi}d\xi. \quad (39)$$

Proof. On the surface $K_\mu^2(t)$, function $|\mathcal{H}(\mathbf{x},t)|$ has con- [redacted] (39) [redacted]

The functions $\Psi(\mu)$ (29) and $\Psi_G(\mu)$ [(35) and (41)] are new invariants of the z-axisymmetric inviscid gas flows inside the mushroom clouds.

DATA AVAILABILITY

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